Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

188. Proposed by SAUL EPSTEEN.

Evaluate $I=\int\dfrac{y(y^2+2)dy}{(y^4+3y^2+3)\sqrt{(y^2+1)}}$.

I. Solution by W. W. BEMAN, M. A., Professor of Mathematics in the University of Michigan.

Putting $y^2+1=z^2$ and $z-1/z=u$, the integrand becomes $\dfrac{1}{u^2+3}$; hence integral $=\dfrac{1}{3^{\frac{1}{2}}}\tan^{-1}\dfrac{1}{3^{\frac{1}{2}}}[(1+y^2)^{\frac{1}{2}}-(1+y^2)^{-\frac{1}{2}}]$.

II. Solution by CHRISTIAN HORNUNG, and M. E. GRABER, Tiffin, Ohio.

Let $y^2+1=u^2$, then $I=\displaystyle\int\frac{(u^2+1)du}{u^4+u^2+1}=\tfrac{1}{2}\int\frac{du}{u^2+u+1}+\tfrac{1}{2}\int\frac{du}{u^2-u+1}$

$$=\frac{1}{3^{\frac{1}{2}}}\left[\tan^{-1}\frac{2(y^2+1)^{\frac{1}{2}}+1}{3^{\frac{1}{2}}}+\tan^{-1}\frac{2(y^2+1)^{\frac{1}{2}}-1}{3^{\frac{1}{2}}}\right]$$

⁂ Dr. G. B. M. Zerr obtains the same result and reduces it to the form

$$\frac{1}{\sqrt{3}}\left[\pi+\tan^{-1}-\frac{(3y^2+3)^{\frac{1}{2}}}{y^2}\right].$$

Also solved by J. Scheffer, E. L. Sherwood, H. M. Armstrong, and G. W. Greenwood. Solved with same result by F. P. Matz.

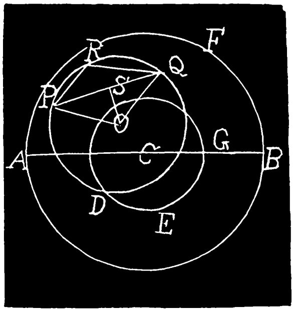
AVERAGE AND PROBABILITY.

130. Proposed by L. C. WALKER, A. M., Santa Barbara, Cal.

Three points are taken at random in a given circle, and a circle passed through them. The probability that the circle through the random points will be wholly in the given circle is $\frac{2}{5}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W Va.

Let PRQ be the triangle formed by joining the three random points P, R, Q in the circle AFB; O the center of the circle circumscribing PRQ. Draw the circle DEG concentric with AFB, both with center C, so that $AD=GB=OP$. Since PQ is known in length and direction, and the angle PRQ is given, if OP is less than AC, the area of DEG represents the number of ways the three points can be taken, so that the circle circumscribing PRQ will lie wholly within AFB.



Let $PQ=2x$, $AC=r$, $\angle POS=\phi$, $\theta=\sin^{-1}(x/r)$, area of segment $PRQ=u$, ρ=angle PQ makes with some fixed line.

Then $PO=x\operatorname{cosec}\phi=r\sin\theta\operatorname{cosec}\phi$, $CD=r-x\operatorname{cosec}\phi=r(1-\sin\theta\operatorname{cosec}\phi)$.

Area $DEG=\pi r^2(1-\sin\theta\operatorname{cosec}\phi)^2$.

$u=x^2(\phi\operatorname{cosec}^2\phi-\cot\phi)=r^2\sin^2\theta(\phi\operatorname{cosec}^2\phi-\cot\phi)$.

An element of surface at Q is $4xdxd\rho=4r^2\sin\theta\cos\theta d\theta d\rho$; at R it is $du=2r^2\sin^2\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)d\phi$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of ϕ, θ and $\pi-\theta$; of ρ, 0 and 2π.

Hence, doubling as R may be on either side of PQ, we get for the required chance,

$$p=\frac{16\pi r^6}{\pi^3 r^6}\int_0^{\frac{1}{2}\pi}\int_\theta^{\pi-\theta}\int_0^{2\pi}\sin^3\theta\cos\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)(1-\sin\theta\operatorname{cosec}\phi)^2d\theta d\phi d\rho$$

$$=\frac{32}{\pi}\int_0^{\frac{1}{2}\pi}\int_\theta^{\pi-\theta}\sin^3\theta\cos\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)(1-\sin\theta\operatorname{cosec}\phi)^2d\theta d\phi$$

$$=\frac{2}{3\pi}\int_0^{\frac{1}{2}\pi}[2(\pi-2\theta)\sin2\theta+4-4\cos4\theta-3\sin^22\theta\cos2\theta$$

$$+64\sin^4\theta\cos\theta\log\tan\tfrac{1}{2}\theta]d\theta=\tfrac{2}{5}.$$

Also solved by F. P. Matz.

Miscellaneous 146 was also solved by Jeannette Brooks.

PROBLEMS FOR SOLUTION.

ALGEBRA.

223. Proposed by **THEODORE L. DELAND**, Office of the Secretary of the Treasury, Washington, D.C.

An officer in the Treasury Department assigned three clerks to count a lot of silver dollars and when finished noted that there was an apparent difference in their efficiency; and, to determine the fact, gave to each a similar lot of the same amount to count, the only record made at the time being that A to count his lot alone, took three weeks longer, B took two weeks longer, and C took one week longer than it took for all working together to count the first lot. The best counter, on the record made, was given an efficiency mark of 93 on the scale of 100. What efficiency mark should, on the record, be given to each of the other two counters?

224. Proposed by **G. W. GREENWOOD, M. A.** (Oxon). Lebanon, Ill.

Show that, if none of the quantities x, y, z is zero, the result of eliminating them from $(x+y)(x+z)=bcyz$ ....(1),

$(y+z)(y+x)=cazx$ ....(2),

$(z+x)(z+y)=abxy$ ....(3),

is $$\begin{vmatrix}\pm a, & 1, & 1\\ 1, & \pm b, & 1\\ 1, & 1, & \pm c\end{vmatrix}=0.$$

[Oxford, 1896.]